but his wife prefers otherwise. In that context, we cannot select (F, O) over (O, F) or vice versa. A similar argument applies in the Chicken game, where player 1 prefers (*Stay*, *Swerve*) to (*Swerve*, *Stay*) while player 2 prefers otherwise.

Risk Dominance

Pareto dominance helps us compare NEs where, for instance, both firms simultaneously change their technology decision from B to A, yielding (A, A). But what if each firm unilaterally deviates from an NE? If (B, B) is played, every firm's payoff loss when unilaterally switching to A is $5 - 0 = 5$; but if (A, A) is being played, every firm losses $10 - 2 = 8$ when unilaterally deviating to B. Informally, at (A, A) unilateral deviations are more risky than at (B, B), as we present below.

Definition 3.5. **Risk dominated NE**. An NE strategy profile $s^* = (s_1^*, s_2^*)$ risk dominates another NE strategy profile $s' = (s_1', s_2')$ if the total payoff loss of moving from s' to s^*,

$$\left[u_1(s_1^*, s_2^*) - u_1(s_1', s_2^*)\right] \times \left[u_2(s_1^*, s_2^*) - u_2(s_1^*, s_2')\right],$$

exceeds that of moving from s^* to s',

$$\left[u_1(s_1', s_2') - u_1(s_1^*, s_2')\right] \times \left[u_2(s_1', s_2') - u_2(s_1', s_2^*)\right].$$

In Matrix 3.11, we can say that (B, B) risk dominates (A, A), because the total payoff loss of moving from (B, B) to (A, A),

$$\begin{aligned} &[u_1(A, A) - u_1(B, A)] \times [u_2(A, A) - u_2(A, B)] \\ &= (10 - 2) \times (10 - 2) = 64, \end{aligned}$$

exceeds that of moving from (A, A) to (B, B),

$$\begin{aligned} &[u_1(B, B) - u_1(A, B)] \times [u_2(B, B) - u_2(B, A)] \\ &= (5 - 0) \times (5 - 0) = 25. \end{aligned}$$

Exercises

3.1. **More general Prisoner's Dilemma game**.^{A} Consider the following Prisoner's Dilemma game where every player simultaneously chooses between confess (F) and not confess (NC). Payoffs satisfy $a_i > b_i > 0$ for every player $i = \{1, 2\}$, implying that everyone earns a higher payoff at (NC, NC) than at (C, C). In addition, $b_i > d_i$ and $c_i > a_i$.

		Player 2	
		C	NC
Player 1	C	b_1, b_2	c_1, d_2
	NC	d_1, c_2	a_1, a_2

a. Find the best responses of each player.
b. Find the NEs of this game.
c. Are your results affected if player 1 enjoys a larger payoff gain from confessing when player 2 does not confess, $c_1 - a_1 > c_2 - a_2$? Interpret.

3.2. **More general Pareto coordination game**.[A] Consider the following Pareto coordination game where every firm simultaneously chooses between technology A or B, and payoffs satisfy $a_i > b_i > 0$ for every firm $i = \{1, 2\}$, implying that both firms regard technology A as superior. However, the premium that firm i assigns to technology A, as captured by $a_i - b_i$, can be different between firms 1 and 2.

		Firm 2	
		Tech. A	Tech. B
Firm 1	Tech. A	a_1, a_2	$0, 0$
	Tech. B	$0, 0$	b_1, b_2

a. Find the best responses of every firm.
b. Find the NEs of this game.
c. Are your results affected if firm 1 assigns a larger premium to technology A than firm 2 does, $a_1 - b_1 > a_2 - b_2$? Interpret.

3.3. **More general anticoordination game**.[A] Consider the following Game of Chicken where every player drives toward each other and simultaneously chooses between swerve and stay. Payoffs satisfy $a_i > b_i > 0$ for every player $i = \{1, 2\}$, implying that drivers suffer more when they both stay (and almost die!) than when they both swerve (and are called chickens), and $a_i > d_i > c_i > b_i$, so being the only player who stays yields a payoff of d_i for that player and $-c_j$ for his rival, where $j \neq i$.

		Player 2	
		Swerve	Stay
Player 1	Swerve	$-b_1, -b_2$	$-c_1, d_2$
	Stay	$d_1, -c_2$	$-a_1, -a_2$

a. Find the best responses of each player.
b. Find the NEs of this game.

c. Are your results affected if the players' payoffs are symmetric, implying that $a_1 = a_2 = a$, $b_1 = b_2 = b$, $c_1 = c_2 = c$, and $d_1 = d_2 = d$?
d. Are your results affected if player 1's payoffs satisfy $a_1 = a_2 + \gamma$, $b_1 = b_2 + \gamma$, $c_1 = c_2 + \gamma$, and $d_1 = d_2 + \gamma$, where parameter γ represents the asymmetry between player 1's and 2's payoffs? Interpret.

3.4. **More general stag hunt game**.[A] Consider the following Stag Hunt Game where two individuals simultaneously choose between hunt a stag or a hare. Payoffs satisfy $a_i > b_i > d_i > c_i$ for every player i, so every player's ranking of payoffs is: (1) hunting a stag with the help of his opponent; (2) hunting a hare with the help of his opponent; (3) hunting a stag without help; and (4) hunting a hare without help.

		Player 2	
		Stag	Hare
Player 1	Stag	a_1, a_2	c_1, d_2
	Hare	d_1, c_2	b_1, b_2

a. Find the best responses of each player.
b. Find the NEs of this game.
c. Are your results affected if players' payoffs are symmetric, implying that $a_1 = a_2 = a$, $b_1 = b_2 = b$, $c_1 = c_2 = c$, and $d_1 = d_2 = d$?

3.5. **Strict dominance and NE**.[B] Consider the following simultaneous-move game between player 1 (in rows) and player 2 (in columns).

		Player 2		
		L	M	R
Player 1	U	$4, 2$	$2, 0$	$0, 3$
	C	$5, 1$	$3, 2$	$\delta, 4$
	D	$5, 2$	$6, 2$	α, β

We are informed that player 1 finds that strategy C weakly dominates U. In addition, we are told that strategy profile (D, R) is a Nash equilibrium but (D, M) is not. Using this information, answer the following questions. Before we start, let us define $u_i(s_i, s_j)$ to be the utility function of player i when player i deploys strategy $s_i \in S_i$ and his counterpart, player j, deploys strategy $s_j \in S_j$, where $i, j = \{1, 2\}$. In this context, $S_1 = \{U, C, D\}$ and $S_2 = \{L, M, R\}$.

a. Does player 2 have a strictly dominated strategy?
b. Is strategy profile (D, R) the unique Nash equilibrium of this game?
c. Does player 1 have a strictly dominant strategy?

3.6. **Rationalizable strategies and IDSDS**.[B] Consider the following two-player game, where player 1 (in rows) chooses U, M, or D, while player 2 (in columns) chooses L or R.

		Player 2	
		L	R
Player 1	U	$4, 2$	$0, 1$
	M	$0, 2$	$4, 1$
	D	$1, 0$	$1, 3$

a. Assuming that players are restricted to use pure strategies, show that IDSDS has no bite. In this context, identify the strategy profiles that survive IDSDS.
b. Allowing now for players to use mixed strategies, show that rationalizability has some bite. (Hint: Use the "never a best response" approach.)

3.7. **Strict dominance and Rationalizability**.[B] Consider Exercise 2.3 again (Chapter 2), where we found the strategy profiles that survive IDSDS and those surviving IDWDS. Find now which strategy profiles survive rationalizability. Compare your equilibrium results against those under IDSDS.

3.8. **IDSDS and rationalizability—Allowing for more general payoffs**.[A] Consider the following simultaneous-move game between Anthony (in rows) and Christine (in columns), where payoff x satisfies $x \geq 0$.

		Christine	
		North	South
Anthony	Up	x, 7	3, 6
	Center	4, 1	9, 2
	Bottom	8, 5	1, 4

a. Identify any strategies that are strictly dominated by pure strategies.
b. Suppose that $x = 6$. Is the strategy *Up* rationalizable? Explain why or why not.
c. Find the minimum value of x for the strategy *Up* to be rationalizable. How are your results affected by an increase in x?

3.9. **NE and rationalizability**.[C] In this exercise, we formally show the relationship between NE strategy profiles and rationalizability.
a. Show that if strategy profile $s^* = \left(s_1^*, \ldots, s_N^*\right)$ is a Nash equilibrium of a N-player game, it must also survive rationalizability.

b. Show that the converse of part (a) is not necessarily true. (For this part, an example suffices.)

3.10. **If a game has an NE, it must survive IDSDS.**[B] Consider the two-player game in the following matrix, where payoffs satisfy $a \neq b \neq c \neq d$. If the game has a unique psNE, show that it must be the unique strategy profile surviving IDSDS.

		Player 2	
		A	B
Player 1	A	a, a	c, c
	B	b, b	d, d

3.11. **A unique strategy profile surviving IDSDS must also be an NE.**[B] Consider that strategy profile $s^* = (s_1^*, s_2^*, \cdots, s_N^*)$ is the unique strategy profile surviving IDSDS in a game with $N \geq 2$ players. Show that s^* must also be the unique NE of the game.

3.12. **Finding NEs in four categories of simultaneous-move games.**[A] Consider the following simultaneous-move game with payoff matrix.

		Player 2	
		A	B
Player 1	A	a, a	b, c
	B	c, b	d, d

Find the players' best responses and the psNEs in the following settings. Interpret and relate your results to common games.

a. Payoffs satisfy $a > c$ and $b > d$.
b. Payoffs satisfy $a > c$ and $b < d$.
c. Payoffs satisfy $a < c$ and $b < d$.
d. Payoffs satisfy $a < c$ and $b > d$.

3.13. **Strictly Dominant equilibrium.**[C] Recall the definition of a strictly dominant equilibrium (SDE) from Sect. 2.8.

a. *Existence.* Provide an example of a game with an SDE and a game without an SDE.
b. *Uniqueness.* Can a game have more than one SDE?
c. Show that if s^{SDE} exists, s^{SDE} must survive IDSDS.
d. Show that if s^{SDE} exists, s^{SDE} must be the only strategy profile surviving IDSDS.
e. Show that if strategy profile s survives IDSDS, it does not need to be an SDE. An example suffices.

3.14. **Strictly Dominant equilibrium and NE.**[C] Consider the discussion about strictly dominant equilibrium (SDE) in Sect. 2.8, where $s^{SDE} = (s_1^{SDE}, s_2^{SDE}, ..., s_N^{SDE})$ denotes an SDE strategy profile.

a. Show that if s^{SDE} exists, then every player has a unique best response to his rivals' strategies.
b. Show that if strategy profile s^{SDE} is a SDE, it must also be an NE.
c. Show that if strategy profile s^{SDE} is a SDE, it must also be the unique NE of the game.

3.15. **Strict Nash equilibrium**.^{B} Consider the following definition: A strategy profile $s^* \equiv (s_1^*, ..., s_N^*)$ is a *strict Nash equilibrium* (SNE) if it satisfies

$$u_i(s_i^*, s_{-i}^*) > u_i(s_i, s_{-i}^*) \text{ for every player } i, \text{ and every } s_i \in S_i.$$

You probably noticed that this definition is almost identical to the definition of Nash equilibrium (NE) strategy profiles, except for using a strict, rather than weak, inequality. In this exercise we connect both solution concepts, but first examine the relationship between a strict Nash equilibrium and IDSDS.

a. Show that a game can have more than one SNE. An example suffices.
b. Provide an example of a game with more than one NE but just one SNE.
c. Provide an example with a game where SNEs are a subset of all strategy profiles surviving IDSDS.
d. Show that if strategy profile s^* is the only strategy profile surviving IDSDS, then s^* must be an SNE.
e. Show that if strategy profile s^* is an SNE, it must also be an NE.

3.16. **Strict and weak Nash equilibria**.^{B} Consider the game in the following payoff matrix.

		Player 2		
		L	C	R
Player 1	U	$6,5$	$7,3$	$8,4$
	M	$4,3$	$10,6$	$5,8$
	D	$5,2$	$11,11$	$4,10$

a. Find the strategy profiles that can be supported as strict Nash equilibria and weak (non-strict) Nash equilibria in this game.
b. Which strategy profiles survive IDSDS?
c. Provide an example of a game with strict and weak Nash equilibria.

3.17. **Split or steal game**.^{A} Consider the following split or steal game. In a TV show, two contestants answer trivia questions, with correct answers increasing a common pot. Once the trivia part of the game is over, assume that the pot accumulates V dollars. Every player is then asked to select independently and simultaneously (from a covered button) whether he wants to split or steal the prize. As described in the following matrix, if both players split the prize, each earns $\frac{V}{2}$; if a player is the only one to steal, he keeps all the

price, V; if he is the only one to split the prize, he earns zero; and if both players steal the prize, both earn zero.[8]

		Player 2	
		Split	*Steal*
Player 1	*Split*	$\frac{V}{2}, \frac{V}{2}$	$0, V$
	Steal	$V, 0$	$0, 0$

a. Does any player have strictly dominated strategies? Does any player have weakly dominated strategies? Explain.
b. Find the NEs of the game.
c. Among all NEs of the game, is some of them strict as defined in Sect. 2.8?

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[8] For more details, you can watch a short video of the TV show here: https://youtu.be/jJzyc6VPMcc.